A Quasi-Closed Form Expression for the Conductor Loss of CPW Lines, with an Investigation of Edge Shape Effects

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Abstract— **In previous work, we used a matched asymptotic technique to investigate the fields near an edge of a finitely conducting strip with nonzero thickness. It was demonstrated that with this asymptotic solution of the fields, the power loss in the region local to the edge could be determined accurately. In this paper, we will show how the accurate representation of the power loss can be used to obtain a closed form expression for the attenuation constant due to conductor loss of coplanar waveguide (CPW) structures. This expression is valid for an arbitrarily shaped edge and any conductor thickness. Results obtained with this expression are compared to and closely agree with both experimental results and other techniques found in the literature. We also investigated conductors with different edge shapes (45° and 90° edges) to explore their effect on the attenuation constant (or loss) of CPW structures.**

I. Introduction

VARIOUS TECHNIQUES have been used to calculate the conductor loss in monolithic microwave integrated circuits (MIMIC's) [1]–[20]. These techniques range from quasi-analytical, like Wheeler's incremental inductance rule, to full numerical approaches, such as mode matching, method of moments (MOM), and finite elements. These numerical techniques are capable of high accuracy but are computationally intensive and hence do not lend themselves to ready use in design. Instead, closed-form expressions are desirable for this purpose.

Traditionally, Wheeler's incremental inductance rule [1] has been used to evaluate the attenuation constant of planar microwave transmission lines, as in the work of Pucel, Massé, and Hartwig [2] for microstrip. For a wide range of applications, this technique works well. However, if the ratio $\frac{t}{\delta}$ is small (where δ is the skin depth and t is the thickness of the conductor) or comparable to 1, then the Wheeler rule breaks down and gives poor results. This is traceable to the fact that the Leontovich surface impedance boundary condition [21], on which the validity of the Wheeler rule depends, is no longer valid. Moreover, if the edges of the strip conductor are not exactly rectangular in shape, but are instead trapezoids with 60° or 70° angles (which often occurs when common fabrication techniques are utilized), then the results of Pucel

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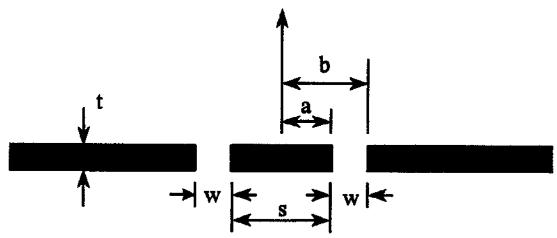

Fig. 1. Geometry of a CPW line.

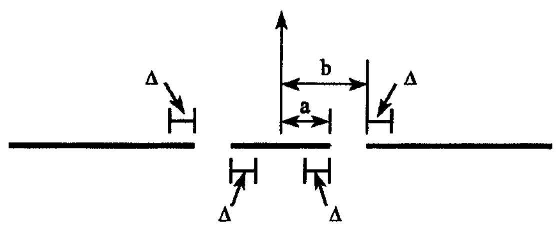

Fig. 2. Geometry of an infinitely thin CPW line.

et al. (obtained assuming a rectangular edge) are also no longer valid. In this case, full numerical techniques are currently the only option for computing the loss.

Consider the CPW (coplanar waveguide) line shown in Fig. 1. It is tempting to determine the conductor loss of this line by using standard perturbation methods for computing wall loss. Assuming that the strips are infinitely thin, an approximate current density on the strips could be obtained and then used to get an expression for the attenuation constant. This expression involves an integral over the top and bottom portions of the strip conductors (Fig. 2)

$$\alpha_c \simeq \frac{R_s}{2Z_o}\int_{\text{top+bottom}}\left(\frac{J}{I}\right)^2 dl \qquad (1)$$

where J is the approximate current density on the strip, I is the total current on the center conductor, R_s is the surface resistance of the strip conductor, and Z_o is the characteristic impedance of the CPW. However, because the current on an infinitely thin, perfectly conducting strip diverges as $1/\sqrt{r}$ (where r is the distance from the nearest edge), the integration of $|J|^2$ would result in a logarithmically divergent integral and this result would be useless.

Lewin [22] and Vainshtein and Zhurav [23] independently developed a method to avoid this difficulty. In the Lewin/Vainshtein procedure, the loss is approximated by carrying out the integration in (1) not to the edges, but to some distance away from the edges of the strip. This distance is chosen so that the resulting power loss near an edge calculated in this way agrees with that of the actual edge. The value of this stopping distance is a function only of the local edge geometry: the strip thickness and the shape of the edge.

In the work by Lewin/Vainshtein and in later work by Barsotti, Kuester, and Dunn [24], the stopping distance (Δ) was determined by evaluating the power loss (with the Leontovich approximation) from an integration of the current distribution (J corresponds to a perfectly conducting strip of the actual edge shape and nonzero thickness) around the contour of the edge. This was then equated to the power loss found from the stopping distance method using the current distribution of a zero thickness perfect conductor. The problem with this approach (which determines the "standard" Lewin/Vainshtein stopping distance) is that because it relies on the Leontovich approximation, it is valid only for skin depths very small compared to strip thickness (that is, at very high frequencies).

In previous work [25] and [26], it has been demonstrated that with a matched-asymptotic expansion technique, a modified Lewin/Vainshtein stopping distance (Δ) can be obtained. This stopping distance can be used for any ratio of strip thickness to skin depth and for any given edge shape, if proper modification to the impedance condition at the strip is made. It was also shown [25] and [26], that once Δ is determined for a given edge shape and as a function of t/δ, a closed-form expression for the conductor loss of a microstrip line could be obtained.

In this investigation, we extend the previous work by deriving a quasi-closed form expression for the attenuation constant of a CPW line. This paper is organized as follows: after the introduction, Section II presents a derivation of the attenuation constant. There, we compare our results to experimental results, numerical results, and to results based on wall loss perturbation approaches. In Section IV, the effect of different edge shapes on the conductor loss is investigated. The last section discusses the ranges within which our quasi-closed form expression is valid.

II. Derivation of the Quasi-Closed Form Expression for Conductor Loss

In [25] and [26], it was demonstrated that with a standard wall-loss perturbation analysis, the change in the propagation constant due to conducting walls for a planar circuit is simply

$$\gamma_m - \gamma_{mo} \simeq -\frac{1}{2Z_o I^2}\left[\int_{\text{top}} \bar{E}^{\text{top}} \cdot \bar{J}^{\text{top}}\, dl + \int_{\text{bottom}} \bar{E}^{\text{bottom}} \cdot \bar{J}^{\text{bottom}}\, dl\right]. \quad (2)$$

If the currents for an infinitely thin conductor are used in the above equation, then the integral will become singular. The Lewin/Vainshtein philosophy says that instead of evaluating the integral out to the edges (where it is singular), the limits of this integral must be taken at some distance just before the edge (the stopping distance Δ). Therefore, (2) is written as

$$\gamma_m - \gamma_{mo} \simeq -\frac{1}{2Z_o I^2}\left[\int_{C_\Delta} \bar{E}^{\text{top}} \cdot \bar{J}^{\text{top}}\, dl + \int_{C_\Delta} \bar{E}^{\text{bottom}} \cdot \bar{J}^{\text{bottom}}\, dl\right] \quad (3)$$

TABLE I
Numerical Results for the Stopping Distance for 90° and 45° Edges

$\frac{t}{2\delta}$	$\frac{t}{\Delta}$ 90°	$\frac{t}{\Delta}$ 45°	$\frac{t}{2\delta}$	$\frac{t}{\Delta}$ 90°	$\frac{t}{\Delta}$ 45°
0.03	9.18	6.57	1.87	266.73	306.15
0.04	9.18	6.57	2.00	244.95	288.35
0.05	9.19	6.57	2.18	221.57	271.18
0.06	9.19	6.58	2.29	210.25	265.33
0.10	9.25	6.62	2.51	200.43	264.15
0.14	9.45	6.77	2.76	189.28	274.11
0.25	11.76	8.43	3.00	178.57	288.74
0.50	33.97	26.72	3.55	170.73	317.89
0.64	61.90	49.41	4.00	168.50	327.08
0.71	81.32	65.51	4.53	171.59	327.42
0.79	108.83	89.23	4.74	172.81	326.99
0.87	138.42	115.75	5.0	174.33	327.01
0.94	169.39	144.36	6.0	185.89	336.93
1.0	200.50	173.95	7.0	193.43	357.86
1.07	235.98	209.06	8.0	195.96	383.06
1.12	258.21	232.17	9.0	196.58	410.96
1.15	270.22	245.19	9.49	196.94	426.04
1.22	299.73	279.65	10.49	198.36	461.40
1.32	324.12	314.30	12.25	203.04	543.07
1.50	329.88	342.89	13.0	205.58	587.90
1.63	312.74	339.00	14.0	209.24	659.63
1.73	293.30	326.77	16.0	217.25	862.14

where C_Δ is the contour of the conducting strip defined to a distance just before the edge. For a CPW line, C_Δ is defined by the following

$$\int_{C_\Delta} = \int_{-a+\Delta}^{a-\Delta} + 2\int_{b+\Delta}^{\infty}$$

where a, b, and Δ are defined in Fig. 2. Δ is the modified Lewin/Vainshtein stopping distance and was determined numerically in [25] for a wide range of strip thicknesses versus skin depths ($\frac{t}{\delta}$) for both a 90° and 45° edge (Table I).

In [25] and [26], it was shown that with an asymptotic technique, a generalized transfer impedance boundary condition that relates the tangential $\bar{E}$ fields on the top and bottom sides of the strips to the currents on the top and bottom sides of the strips could be given by

$$\left.\begin{aligned} \bar{E}_{\tan}^{\text{top}} &= (j\omega\mu_o \tfrac{t}{2} + Z_s)\bar{J}^{\text{top}} + Z_m \bar{J}^{\text{bottom}} \\ \bar{E}_{\tan}^{\text{bottom}} &= (j\omega\mu_o \tfrac{t}{2} + Z_s)\bar{J}^{\text{bottom}} + Z_m \bar{J}^{\text{top}} \end{aligned}\right\} \quad (4)$$

where

$$Z_s = -j\sqrt{\frac{\mu_o}{\epsilon_c - j\frac{\sigma_c}{\omega}}}\cot(k_c t)$$

$$Z_m = -j\sqrt{\frac{\mu_o}{\epsilon_c - j\frac{\sigma_c}{\omega}}}\csc(k_c t)$$

and k_c, σ_c, and ϵ_c are the wave number, the conductivity, and the permittivity of the conductor, respectively.

The final goal of this analysis was to obtain the attenuation constant of a CPW line given by

$$\alpha_m = \mathrm{Re}(\gamma_m - \gamma_{mo}).$$

The first term in the boundary condition given in (4) can be neglected because it is a purely imaginary number and will contribute nothing to the power loss. With the exception of the term $j\omega\mu_o t/2$, this equation is equivalent to the generalized transfer impedance boundary condition given by Horton [6]. The extra term reflects the extrapolation of the asymptotic solution from points on the actual conductor surface ($y = \pm t/2$) to the fictitious half-plane $y = 0$ (see [25] and [26] for more details).

Instead of working with both the top and bottom currents, it is preferable to express the top and bottom currents as one-half the total current ($\bar{J}$), plus or minus a difference current ($\delta\bar{J}$)

$$\left.\begin{array}{rcl} \bar{J}^{\mathrm{top}} &=& \frac{1}{2}\bar{J} - \delta\bar{J} \\ \bar{J}^{\mathrm{bottom}} &=& \frac{1}{2}\bar{J} + \delta\bar{J} \end{array}\right\}. \tag{5}$$

If (4) and (5) are substituted into (3), the following is obtained

$$\left.\begin{array}{l} \gamma_m - \gamma_{mo} \\ \simeq \frac{Z_s+Z_m}{4Z_o}\int_{C_\Delta}\left(\frac{\bar{J}\cdot\bar{J}}{I^2}\right)dl + \frac{Z_s-Z_m}{2Z_o}\int_{C_o}\left(\frac{\delta\bar{J}\cdot\delta\bar{J}}{I^2}\right)dl \end{array}\right\}. \tag{6}$$

Note that the second integral can be extended over all of C_o (including the edge) because the integral is not singular at the edges.

The real part of (6) represents the attenuation constant of the CPW line. For quasi–TEM CPW lines (i.e., when the dimensions of the gaps and of the center conductor are small compared to a wavelength), the current distribution on the strips is governed by a magnetostatic problem and is independent of any dielectric that is present. The top and bottom currents for this case are identical and δJ can be neglected. Thus, for a quasi–TEM CPW line, the attenuation constant reduces to the following

$$\alpha \simeq \frac{R_{sm}}{4Z_o}\int_{C_\Delta}\left(\frac{J}{I}\right)^2 dl \tag{7}$$

where J is the current density on the planar circuit, and R_{sm} is the resistance in the modified Horton impedance boundary condition discussed in [25] and [26] and given by

$$R_{sm} = \omega\mu_c t\,\mathrm{Im}\left(\frac{\cot(k_c t) + \csc(k_c t)}{k_c t}\right). \tag{8}$$

If the current distribution (J) of a planar circuit is known, then the attenuation constant for that structure can be determined through (7). Equation (7) was used in [25] to determine the conductor loss for a microstrip line. Here, the attenuation constant given in (7) will be employed to determine the loss for CPW lines. The current distribution for the CPW structure shown in Fig. 2 is given in [16]. On the central strip

$$J = \frac{A}{\sqrt{(a^2-x^2)(b^2-x^2)}} \quad ; \quad |x| < a \tag{9}$$

where

$$A = \frac{bI}{2K(k)} \quad ; \quad k = \frac{a}{b}$$

and $K(k)$ is the elliptic integral of the first kind. On the ground planes

$$J = -\frac{A}{\sqrt{(x^2-a^2)(x^2-b^2)}} \quad ; \quad |y| > b. \tag{10}$$

The characteristic impedance for a CPW line is

$$Z_o = \frac{\zeta_o}{4\sqrt{\epsilon_{eff}}}\frac{K(k')}{K(k)}$$

where $k' = \sqrt{1-k^2}$. If $h >> b$ (where h is the distance to the bottom ground plane of the CPW line), then ϵ_{eff} can be approximated by

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2}.$$

Once the current distribution given above is substituted into (7), the following is obtained

$$\alpha \simeq \left.\begin{array}{l} \frac{\mathrm{Re}(Z_s+Z_m)}{4Z_o}\left\{\frac{s}{I^2}\int_0^{a-\Delta}\frac{A^2\,dx}{(a^2-x^2)(b^2-x^2)}\right. \\ \left. +\frac{s}{I^2}\int_{b+\Delta}^{\infty}\frac{A^2\,dx}{(x^2-a^2)(x^2-b^2)}\right\} \end{array}\right\}. \tag{11}$$

The integrals are elementary and give

$$\alpha \simeq \left.\begin{array}{l} \frac{R_{sm}b^2}{16Z_oK^2(k)(b^2-a^2)}\left\{\frac{1}{a}\ln\left[\left(\frac{2a}{\Delta}-1\right)\left(\frac{b-a+\Delta}{b+a+\Delta}\right)\right]\right. \\ \left. +\frac{1}{b}\ln\left[\left(\frac{2b}{\Delta}+1\right)\left(\frac{b-a+\Delta}{b+a-\Delta}\right)\right]\right\} \end{array}\right\}. \tag{12}$$

If Δ is small compared to a, b, and $(b-a)$, then

$$\alpha \simeq \frac{R_{sm}b^2}{16Z_oK^2(k)(b^2-a^2)} \cdot\left\{\frac{1}{a}\ln\left(\frac{2a}{\Delta}\frac{b-a}{b+a}\right) + \frac{1}{b}\ln\left(\frac{2b}{\Delta}\frac{b-a}{b+a}\right)\right\}. \tag{13}$$

This result can qualitatively be compared to some results in the literature. Owyang and Wu [16] analyzed the loss of a CPW structure with an air dielectric, $\epsilon_r = 1$. To check our results with [16], the Lewin/Vainshtein value for Δ (high frequency limit) was used for a rectangular edge

$$\Delta_{LV} = \frac{t}{4\pi e^\pi} = \frac{T}{2\pi e^\pi}$$

so

$$\alpha_{LV} \simeq \left.\begin{array}{l} \frac{R_{sm}b^2}{16Z_oK^2(k)(b^2-a^2)}\left\{\frac{1}{a}\left[\pi + \ln\left(\frac{4\pi a(b-a)}{T(b+a)}\right)\right]\right. \\ \left. +\frac{1}{b}\left[\pi + \ln\left(\frac{4\pi b(b-a)}{T(b+a)}\right)\right]\right\} \end{array}\right\}. \tag{14}$$

This is exactly twice (36) in [16]; we believe this is due to an error or misprint in [16]. Comparisons in Section III suggest that our expression is correct.

III. Comparison to Experimental and Other Results

The loss predicted by (13) can be compared both to experimental results and to full numerical results. Fig. 3 shows results obtained from equation (13) for a CPW structure for a wide frequency range with $t = 3.0$ μm, $a = 5$ μm, $b =$

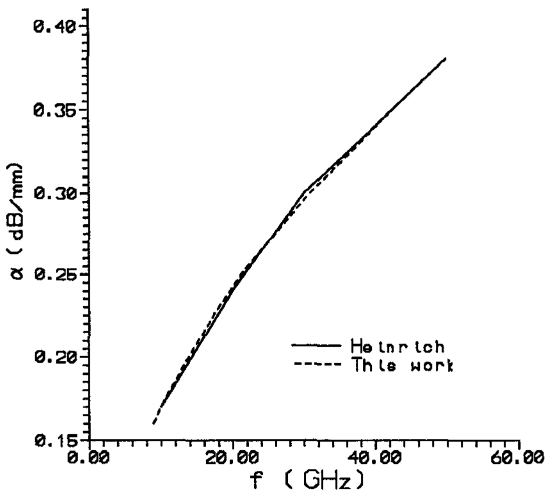

Fig. 3. Comparison of predicted loss with Heinrich's theory for a CPW line with $t = 3.0$ μm, $a = 5$ μm, $b = 25$ μm, $\epsilon_r = 12.9$, and $\sigma_c = 3.0 \cdot 10^7$.

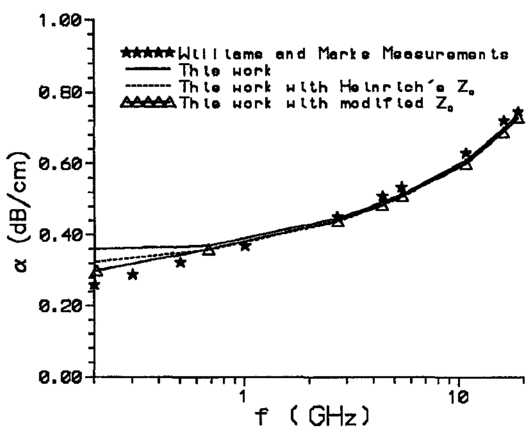

Fig. 4. Comparison of predicted loss with experimental values for a CPW line with $t = 1.61$ μm, $a = 35.6$ μm, $b = 84.6$ μm, $\epsilon_r = 12.9$, and $\sigma_c = 3.602 \cdot 10^7$.

25 μm, $\sigma_c = 3.0 \cdot 10^7$. Also plotted on this figure are results Heinrich [5] obtained using a mode-matching technique. The two different theories correlate very well.

Fig. 4 shows results for a CPW structure with $t = 1.61$ μm, $a = 35.6$ μm, $b = 84.6$ μm, $\sigma_c = 3.602 \cdot 10^7$, and $\epsilon_r = 12.9$. Also shown are the experimental values that Williams and Marks [27] obtained for the same structure. This curve shows excellent agreement with the experimental results except for very low frequency values $f < 0.5$ GHz. This is explained by noting that the characteristic impedance used in (13) is not valid for low frequency. In this analysis we have assumed that the characteristic impedance obtained is that of a pure TEM mode. At the low frequencies, fields penetrate significantly into the conductors, and the mode is no longer close to that when the conductors are perfect. By modifying the characteristic impedance (Z_o), the low frequency discrepancy can be improved.

Williams and Marks [27] supplied us with characteristic impedances that were obtained from the Heinrich mode matching code. These values of Z_o were used in (13), and the results

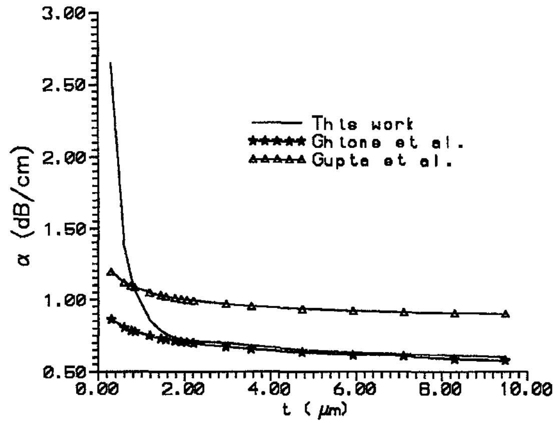

Fig. 5. Variations of loss as a function of conductor thickness with $a = 35.6$ μm, $b = 84.6$ μm, $\epsilon_r = 12.9$, and $\sigma_c = 3.602 \cdot 10^7$.

are shown in Fig. 4. The low-frequency results now correlate well with the experimental results. However, the goal here is to have a theory that does not require numerically determined characteristic impedance for the structure.

The expression given in (13) is valid as long as the frequency is not too low. The problem at low frequency is that the characteristic impedance is perturbed by the presence of the magnetic fields that penetrate the conductor. In [26] and [28], closed-form corrections to the characteristic impedance for a microstrip and CPW line were developed. These corrections are based on the attenuation constants derived in this paper and in [25]. From [26], it is shown that the attenuation constant for a corrected characteristic impedance is given by

$$\alpha \simeq \mathrm{Re}\left(\frac{F}{Z_o}\frac{2}{1+\sqrt{1+\frac{4F}{Z_o\gamma_{mo}}}}\right) \tag{15}$$

where

$$F = \frac{Z_s + Z_m}{4}\int_{C_\Delta}\left(\frac{J}{I}\right)^2 dl$$

or

$$F = \frac{(Z_s+Z_m)b^2}{16K^2(k)(b^2-a^2)}\left\{\frac{1}{a}\ln\left(\frac{2a}{\Delta}\frac{b-a}{b+a}\right)+\frac{1}{b}\ln\left(\frac{2b}{\Delta}\frac{b-a}{b+a}\right)\right\}$$

and $\gamma_{mo} = jk_o\sqrt{\frac{\epsilon_r+1}{2}}$. Z_s and Z_m are defined earlier.

Fig. 4 compares the loss predicted by this equation with the experimental values obtained in [27]. From this figure it is seen that for the high-frequency end this new, modified value for α correlates well with the previous prediction. At low frequencies, (15) compensates for the incorrect impedance used in (13).

Using the same CPW line as was used in the last example ($a = 35.6$ μm, $b = 84.6$ μm, $\sigma_c = 3.602 \cdot 10^7$, and $\epsilon_r = 12.9$) the effect of conductor thickness on loss was investigated. Fig. 5 shows results for this CPW structure for a frequency of 20 GHz and for various values of t (the conductor thickness).

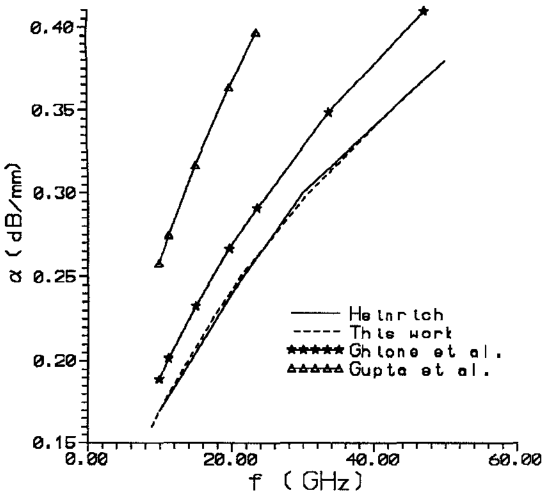

Fig. 6. Comparison of predicted loss with other expressions found in the literature for a CPW line with t = 3.0 μm, a = 5 μm, b= 25 μm, ϵ_r = 12.9, and σ_c = 3.0 · 10^7.

As expected, the loss increases very rapidly once the conductor thickness becomes less than about 2δ–3δ ($\delta = 0.593$ μm).

Also shown in this figure are results from two different closed form expressions for the loss found in the literature. The first expression is based on a wall loss perturbation procedure in which the current density on a conductor with finite thickness (obtained from conformal mapping) is utilized. This expression was first introduced by Owyang and Wu [16] and then later modified by Ghione, Naldi, and Zich [19], and can also be found in Wadell [29]. The second expression is based on the Wheeler's incremental inductance rule and is given in Gupta, Garg, and Bahl [30].

Figs. 5 and 6 show a comparison of these two expressions to our results along with numerical results. These figures show that the results based on the Ghione, Naldi, and Zich [19] procedure predict losses differing by 12–30% from those obtained from both our model and numerical results. Even more deviation is seen when $\frac{t}{\delta}$ is small. This deviation is due in part to the fact that the Leontovich surface impedance is no longer valid for small $\frac{t}{\delta}$, nor is it valid in the vicinity of the edge.

Figs. 5 and 6 also illustrate that the results based on Wheeler's incremental inductance rule (see Gupta *et al.* [30]) deviate from our results as well as from the results based on the Owyang and Wu technique. The drastic deviation in the results obtained from Wheeler's rule is probably traceable to the derivatives of Z_o needed in the Gupta *et al.* [30] formula. The formulas for Z_o are accurate to a few percent, but this assertion says nothing about the accuracy of derivatives of Z_o, and therefore caution must be used when employing values of $\frac{\partial Z_o}{\partial n}$ computed in this way.

IV. Edge Shape Effects on Conductor Loss

The fabrication process used in manufacturing this structure can result in edge profiles other than 90°. Various different edge profiles could be analyzed to determine how conductor loss is affected by edge shape. By considering a 45° edge as a worst case scenario, the change in loss due to edge shapes can be gauged. Figs. 7 and 8 show the results of two CPW lines with 45° edges on both the central and outer strips. The loss for the 45° edge is not significantly more than the loss of the 90° edge when t = 1.61 μm (Fig. 8). When t = 3 μm, there is about a 7% increase in the loss when f = 40 GHz (Fig. 7). Therefore, for thick CPW lines, the additional loss associated with the edge may indeed be significant.

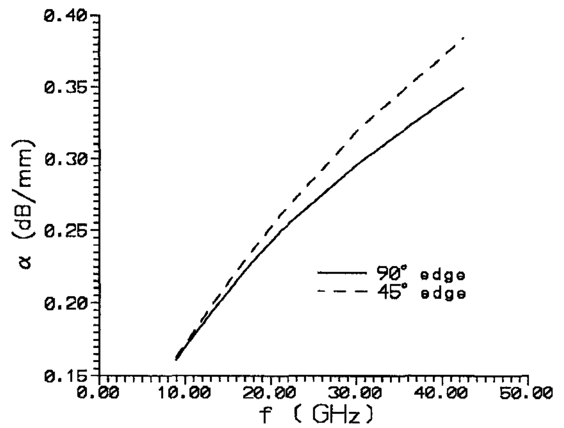

Fig. 7. Comparison of the predicted loss for both 90° and 45° edges with t = 3 μm, a = 5 μm, b = 25 μm, ϵ_r = 12.9, and σ_c = 3.0· 10^7.

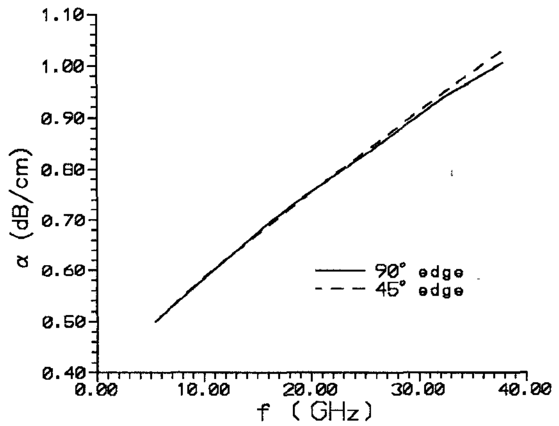

Fig. 8. Comparison of the predicted loss for both 90° and 45° edges with t = 1.61 μm, a = 35.6 μm, b = 84.6 μm, ϵ_r = 12.9, and σ_c = 3.602· 10^7.

V. Range of Validity of the Loss Expression

In this section we will discuss the scenarios where the expression presented here is valid. We begin by discussing the effect of thickness, edge shape, and skin depth. In developing the stopping distance (Δ), an asymptotic expansion of the fields local to the edge was used to accurately characterize the power loss in the vicinity of an arbitrarily shaped edge. This results in a stopping distance, and, more important, in a loss expression, that is valid for any strip thickness to skin depth ratio ($\frac{t}{\delta}$). The stopping distance is also valid for any edge shape one chooses to analyze. In this paper we show results for both a 90° and a 45° edge.

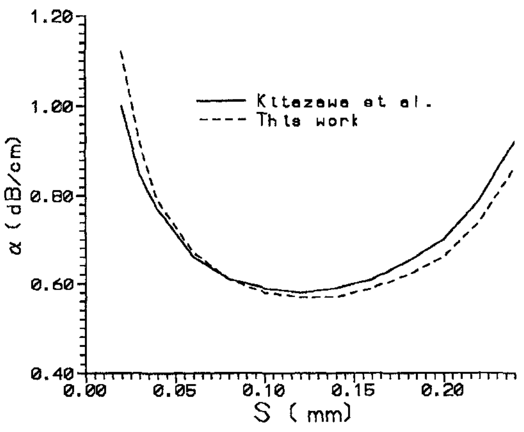

Fig. 9. Variation of loss as a function of center conductor width with $t = 3.0\ \mu\text{m}$, $b = 150\ \mu\text{m}$, $\epsilon_r = 12.8$, and $\sigma_c = 5.882 \cdot 10^7$. The dashed curve shows our results, and the solid curve is from Kitazawa and Itoh [17].

As long as the frequency is not too low (so that the quasi–TEM mode Z_o can be used), then (13) does an excellent job of predicting the loss of a CPW line. At the low frequencies, the magnetic field penetrates the conductors, and this causes the quasi–TEM mode Z_o to be invalid. Once this occurs, a correction to Z_o is needed. Such a correction has been applied here [see (15)], and good agreement for the loss at low frequencies was shown.

The main limitation of the work presented here is the manner in which the stopping distance was derived. In deriving the stopping distance ([25] and [26]) it was assumed that the edge of the strip was isolated from other strip edges. Therefore, if the center conductor of a CPW line is too narrow or if the gap between the center and outer conductor is too narrow, then the validity of the stopping distance and the expression for loss are in question.

This point can be illustrated by investigating the loss of a CPW line with a fixed value of b (Fig. 1) and various values of a (the half-width of the center conductor). In Fig. 9, we compare our results to results obtained from a so-called hybrid-mode formulation [17] for various values of a. This figure shows that when either a approaches b ($\frac{t}{b-a} \rightarrow .1$, in this example) or when a gets very small ($\frac{t}{a} \rightarrow .3$, in this example), our results deviate from the MOM results by only 5–10%, which is quite good considering the assumptions made in deriving the stopping distance.

In principle, it should be possible to obtain a new stopping distance (Δ), valid for closely spaced edges by modifying the approach of [25]. This will be the topic of future work.

VI. Conclusion

In this paper, we have developed a closed-form expression for the conductor loss of a CPW line. This expression is valid for any strip thickness-to-skin depth ratio and any edge shape. Results from this expression have been compared to both experimental results and to results obtained from a full numerical approach, and excellent correlation was demonstrated. Comparisons to other expressions for the conductor loss, obtained from either Wheeler's rule or from a perturbation procedure, have been made and the limitation of these other expressions is given.

The expression presented here is based on knowing the characteristic impedance of the structure being analyzed. Using a TEM assumption for Z_o gives very good results for most frequencies. Once the frequency becomes very low, corrections to Z_o are needed to take into account the magnetic fields penetrating the conductors. Such an expression is given here and is shown to correlate well with experimental data.

We have investigated the effects of different edge shapes on the conductor loss of a CPW line. It was shown that for large $\frac{t}{\delta}$, the additional power loss associated with 45° may indeed be significant.

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